Comparative Analysis between Response-Factor and Finite-Volume based Methods for predicting Heat and Moisture Transfer through Porous Building Materials

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ABSTRACT: Many of the now well-known building energy simulation programs use the response factor method developed in the early 70’s by Stephenson and Mitalas. These are TRNSYS, EnergyPlus, Blast and DOE-2, to name a few of them. Others, such as PowerDomus, ESP-r and BSim, perform finite-volume or finite-difference calculations to solve the heat and mass transfer through the building envelope. These two different approaches are known to have strength and weakness. The main objective of the present exercise is to compare the prediction of both methods. A two-step procedure is employed here. The first deals with pure thermal problem, i.e., without moisture calculation. Three different cases of increasing complexity are studied and compared to analytical solutions. The second step focus on the moisture problem alone by comparing the responses obtained with a two-layer buffer storage model and a finite-volume discretization for moisture transfer. Results show that time step values are determinant even for pure thermal cases where the classical value of 1 hour can lead to notable errors. For problems with moisture sorption in the wall, it has been shown that grid refinement is a very decisive parameter, while time step has to be set to unusual small values to achieve a good response.

KEY WORDS: heat, moisture, response factor, finite-volume, envelope, building.

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INTRODUCTION

Response-factor based methods (BES-1) are used in most building energy simulation softwares because they accurately predict heat transfer in buildings with small computational time. Moisture transfer needs to be evaluated too in order to take into account the heat generation induced by moisture exchanges between the air and the building envelope and its effect on the relative humidity level of the zone air required to predict thermal comfort. As a consequence, simplified models describing moisture transfer at wall have been implemented in BES-1. With the increase of computational resources, volume-finite based methods (BES-2) for building energy simulation have been developed in the past few years. They are more accurate than BES-1 as they model heat and moisture transfer through walls without the simplifications involved in BES-2.

As stated by Judkoff (1988), there are three ways to evaluate the accuracy of these simulation programs: empirical validation in which calculated results are compared to monitored data from a real building or test cell such as PASSYS (1993) and ETNA (1999), analytical verification in which outputs from a program are compared to results from a known analytical solution (ASHRAE Test Suite, 2001), and comparative testing in which a program is compared to itself or to other programs such as BESTEST (1995), HVAC BESTEST (2002) and HAMSTAD (2002).

The present study lies on the second way of evaluation as a comparison exercise between response-factor and finite-volume methods. This exercise aims to compare these two approaches with analytical solution for very simple and highly constrained boundary conditions of heat and moisture transfer through walls. Two softwares have been used to perform the calculations. They can be considered as two representative examples of the actual building energy simulation programs because the first one uses the response-factor method with a simplified moisture model and the second one employs the finite-volume method for both the heat and moisture transfer through the building envelope. In a first part, the main theories, methodologies, capabilities and limitations of both softwares are briefly reported. Three cases of heat transfer and two cases of moisture transfer are then described. Results are presented and discussed in a third part.
THEORY AND METHODOLOGY

The classical partial equation describing conduction heat transfer based on Fourier’s law and its general boundaries conditions are given by equations (1) and (2).

\[
\frac{\partial T}{\partial t} = -a \frac{\partial^2 T}{\partial x^2}, \quad \text{and} \\
q = -\lambda \frac{\partial T}{\partial x} \quad \text{at the wall boundaries.}
\]  

(1)  

(2)

In building simulation softwares, this equation needs to be solved in conjunction with many building processes such as heating, ventilation and air conditioning among others. In sake of clarity, only the solution of heat and moisture transfer through the building envelope is presented here. Specific procedures are briefly introduced in each numerical method.

BES-1

BES-1 heat transfer calculations are based on the transfer function methodology that has been introduced by Stephenson and Mitalas (1971). The transfer function links the present surface heat flux to the past and present surface temperature and heat flux. The relationships of Mitalas and Arseneault (1971) are used in BES-1:

\[
q_{si} = \sum_{j=0}^{n_s} b^j T_{so} - \sum_{j=0}^{n_s} c^j T_{si} - \sum_{j=1}^{n_s} d^j q_{si}, \quad \text{and} \\
q_{so} = \sum_{j=0}^{n_s} a^j T_{so} - \sum_{j=0}^{n_s} b^j T_{si} - \sum_{j=1}^{n_s} d^j q_{so}.
\]  

(3)  

(4)

where \(j\) refers to the term in the time series (current time \(j = 0\), previous time \(j = 1\)…). The time base on which these calculations are based is specified by the user. The time base value has to be lower than the thermal time-constant of the wall and higher than the transfer function algorithm stability criteria. Typical values for building walls are within 0.25 to 2 hours. The coefficients of the time series (a, b, c and d) are determined from the material layers properties of the wall (i.e. thermal conductivity, capacity, density and thickness) using the z-transfer function algorithm presented in Mitalas and Arseneault (1971). A comparison of other methods used to find these coefficients can be found in Iu and Fisher (2004). It should be pointed out that in BES-1, the calculation of these coefficients is performed only once at the beginning of the simulation. As a consequence, variable material properties induced for
example by variable moisture content in the material, cannot be taken into account in the heat transfer calculation.

As already stated, as heat and moisture transfer within the building envelope are decoupled in BES-1, the two available models for moisture calculation do not consider the thermal gradient within the wall. The first one considers sorption effects at walls with an enlarged moisture capacity of the zone. The main problem with this model is to correctly evaluate the new air capacity of the zone air based on the materials that compose the envelope. The second, more sophisticated model, is the so-called Buffer Storage Humidity Model. This model describes a separate humidity buffer divided into a surface and a deep storage portion. Each buffer is defined by three parameters: the gradient of the sorptive isothermal line of the material ($\zeta$), the mass of the material ($M$) and the moisture exchange coefficient between the two regions ($\beta$) and the zone air (Figure 1).

![Figure 1: Buffer Storage Humidity Model.](image)

The two following differential equations describe the dynamics of the water content of the surface and the deep storage.

\[
M_{\text{surf}} \zeta_{\text{surf}} f(\varphi, \omega) \frac{d\omega_{\text{surf}}}{dt} = \beta_{\text{surf}} (\omega_{\text{in}} - \omega_{\text{surf}}) + \beta_{\text{deep}} (\omega_{\text{deep}} - \omega_{\text{surf}}) \tag{5}
\]

\[
M_{\text{deep}} \zeta_{\text{deep}} f(\varphi, \omega) \frac{d\omega_{\text{deep}}}{dt} = \beta_{\text{deep}} (\omega_{\text{surf}} - \omega_{\text{deep}}) \tag{6}
\]

The difficulty of this simple model relies on the determination of the mass of the material of each wall regions which depends on the value of the effective moisture penetration depth $d_{\text{EMPD}}$. This penetration depth can be evaluated from the vapor...
diffusion coefficient in the porous media and the period of the moisture perturbation (Cunningham, 1992):

\[ d_{\text{EMPD}} = \sqrt{\frac{D_{\text{v}} \times \tau}{\pi}}. \]  

(7)

It should be noticed that this model allows taking into account the moisture transfer between external walls, internal walls or furniture and the zone air only. No moisture transfer with the exterior is modeled; as a result the outdoor climate has no effect on the moisture transfer within the building envelope.

**BES-2**

BES-2 calculates the coupled heat and moisture transfers through the building walls. The governing partial differential equations derived from conservation of energy and mass flow in an elemental volume of porous material are presented below.

\[
\rho c_m(T, \theta) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda(T, \theta) \frac{\partial T}{\partial x} \right) - L_{\text{vap}}(T) \rho_{h, \text{O}} \frac{\partial}{\partial x} \left( D_{\text{v}}(T, \theta) \frac{\partial \theta}{\partial x} \right), \quad \text{and} \quad (8)
\]

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left( D_{\text{v}}(T, \theta) \frac{\partial \theta}{\partial x} \right). \quad (9)
\]

Note that the first equation differs from Fourier’s equation for transient heat flow by an added convective transport term (due to moisture diffusion associated with evaporation and condensation of water in the pores of the medium) and by a dependence on the moisture content (so that it is coupled to the second equation). The driving forces for convective transport are temperature and moisture gradients. Nevertheless, vapor diffusion is normally written in the literature in terms of a vapor pressure gradient and data are provided for the water vapor permeability. The vapor transport coefficient is evaluated from the following relation:

\[ D_{\text{v}} = \delta_p \frac{p_{\text{sat}}}{\zeta} \frac{1}{\rho} \]  

(10)

The liquid transport coefficients are based on capillary pressure data, which are ignored in the present study.

The governing partial differential equations are discretized using a fully-implicit scheme and the equations system is solved by means of the Multi-Tri-Diagonal Matrix Algorithm (MTDMA) by Mendes et al. (2002) that was shown to be more stable, accurate and time-efficient than the Tri-Diagonal Matrix Algorithm (TDMA).
Simulation Parameters

In both methods, the precision of the calculations can be adjusted via two parameters. In reality, there are also the convergence parameters of the different iterative calculations but, in this study, these parameters have been set constant and small enough to not represent a potential source of errors.

In BES-1, the first parameter is the time base that is used to determine the transfer function coefficients. The second parameter is the time step of the simulation. The time step has to be lower than or equal to the time base. If it is lower, a linear interpolation is performed which can cause small errors. As a consequence, it is always better to keep the same value for these two parameters. This fact limits the range of possible time step values for a particular wall. Here, the time step is always chosen equal to the time base value. In BES-2, the parameters are the number of nodes used to spatially discretize the wall thickness and the time step of the simulation. Even if the method is stable, the values of both parameters have to be set small enough to reach the solution. No specific rule exists to determine the right values of these parameters; so they have to be adjusted for each case.

If not specified, the time step is set to 60 min for heat transfer for both methods and 15 min and 1 min for moisture transfer for BES-1 and BES-2 respectively. The finite-volume length is set to 1 mm for heat and moisture transfer. These values can be considered as usual values to perform building energy simulations.

DESCRIPTION OF THE STUDIED CONFIGURATIONS

The BESTEST (1995) building geometry without windows is chosen as a reference for all cases. Its internal dimensions are 8 m × 6 m × 2.7 m. Convective heat transfer coefficients are 3.2 W.m⁻².K⁻¹ at the internal wall surface and 24.7 W.m⁻².K⁻¹ at the external one. Moisture transfer is only possible at the internal surfaces and the surface transfer coefficient for vapor concentration is \( \beta_c = 2.71 \times 10^{-3} \) m.s⁻¹.

Thermal Transfer through the Building Envelope

The three basic cases of thermal perturbation (Figure 2) are based on the ASHRAE Test Suite (2001) that provides analytical solutions (Appendices A, B and C) for an external step perturbation with an adiabatic internal wall surface (TC1 as Transient Conduction case 1), an external step perturbation with a constant zone air temperature (TC2) and an external sinusoidal perturbation with a constant zone air temperature (TC3). The external temperature values are \( T_0 = 20 \) °C, \( T_1 = 35 \) °C and \( T_{\text{amp}} = 7.5 \) °C.
Figure 2: Thermal perturbations of tests TC1, TC2 and TC3.
Two materials (Table 1) and two thicknesses (\(L = 0.04\) m and \(L = 0.25\) m) were chosen in order to test the range of thermal inertia encountered in real buildings.

<table>
<thead>
<tr>
<th>Material</th>
<th>(\lambda) (W.m(^{-1}).K(^{-1}))</th>
<th>(\rho) (kg.m(^{-3}))</th>
<th>(c_m) (J.kg(^{-1}).K(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>1.13</td>
<td>1400</td>
<td>1000</td>
</tr>
<tr>
<td>Insulation</td>
<td>0.04</td>
<td>10</td>
<td>1400</td>
</tr>
</tbody>
</table>

Moisture Transfer through the Building Envelope

Two basic cases of moisture perturbation (Figure 3) have been studied here. Both configurations are isothermal (\(T = 20^\circ\)C). The first case is an internal moisture step perturbation with an impermeable external wall surface (MT1 for Moisture Transfer case 1). The analytical solution has been deduced by the present authors considering the complete analogy with the TC1 problem.

The solution is based on the one-dimensional unsteady moisture diffusion equation:

\[
\frac{\partial c_w(x,t)}{\partial t} = a_c \frac{\partial^2 c_w(x,t)}{\partial x^2}.
\] (11)

The boundary conditions are:

\[
c_w(x,0) = c_{in}(0) = c_{out}(0) = c_0 ,
\] (12)

\[
c_{out}(t > 0) = c_1 ,
\] (13)

\[
\frac{\partial c_w(x = L, t > 0)}{\partial x} = 0 , \text{ and}
\] (14)

\[
\beta_c (c_w(x = 0,t) - c_{in}) = -\delta_c \frac{\partial c_w(x = 0,t)}{\partial x} .
\] (15)

where \(a_c = \delta_c c_{sat} \zeta \), \(\beta_c = \beta RT\) and \(\delta_c = \delta_p RT\).

The solution is obtained by analogy with the solution of Appendix A:

\[
c_w(x,t) = c_0 + (c_0 - c_{in}) \sum_{n=1}^{\infty} a_n e^{-\alpha_i^2 Fo} \cos \left( \alpha \frac{L-x}{L} \right) .
\] (16)
where \( \alpha \tan \alpha = \text{Bi} \), \( \text{Bi} = \frac{\beta_c L}{\delta_c} \) and \( \text{Fo} = \frac{a_c t}{L^2} \).

For the present case, the vapor concentration values are \( c_0 = 5.22 \text{ g.m}^{-3} (= 30 \% \text{ of relative humidity at } 20 \degree \text{C}) \) and \( c_1 = 13.92 \text{ g.m}^{-3} (= 80 \%). \)

The second case (MT2) is a periodic moisture perturbation induced by the presence of an internal moisture gain \( (G_0 = 0.5 \text{ kg.h}^{-1}) \) between 9:00 and 17:00 every day. There are no moisture gains outside these hours. External air enters the room at a constant rate of 0.5 ach. Outside and initial indoor vapor concentrations are set to \( c_0 \). These are also the initial conditions of the material. The external wall surface is impermeable. A semi-analytical solution has been provided by Bednar and Hagentoft (2005). The appendix D presents the main steps of the solution.
For both cases, the wall is made up of 0.15 m aerated concrete ($\rho = 650 \text{ kg.m}^{-3}$). The vapour permeability is $\delta_p = 3.0 \times 10^{-11} \text{ kg.m}^{-1} \text{s}^{-1} \text{Pa}^{-1}$ and the moisture capacity of the material is $\zeta = 0.0661 \text{ kg.m}^{-3}$.

HEAT TRANSFER RESULTS

Test TC1: Transient Conduction – Step response

![Graphs of temperature evolution for TC1 configuration](image)

Figure 4: Temperature evolution for the TC1 configuration (TSI: inside surface temperature, TSO: outside surface temperature).

The external surface is exposed to an environment at 35 °C while the interior surface is adiabatic. Figure 4 presents the temperature evolution for the four walls. On the whole, the two programs give the same response. They are both very close to the
analytical solution with greater differences obtained with BES-1 right after the temperature step for the thickest walls. For the smallest Biot number case (0.04 m concrete wall), both programs completely miss the temperature increase. Additional simulations have been performed to test the temperature response versus the simulation time step (Figure 5). BES-1 seems to react faster than BES-2 to a time step decrease but it seems that a limit exists for BES-1 under which it tends to overestimate the temperature and to increase the error. At the opposite, the smaller the time step is, the higher the accuracy is for BES-2.

![Figure 5: Temperature evolution for the TC1 configuration (concrete wall)](image)

**Test TC2: Transient Conduction – Step response**

The external surface is exposed to an environment at 35 °C while the interior zone stays at 20 °C. Figure 6 presents the temperature evolution for the four walls. Again, the two programs give similar responses. They are both very close to the analytical solution except for the 0.04 m concrete wall. Figure 7 presents the additional simulations for this case for the inside surface temperature, the same trend is observed for the outside surface temperature. Same conclusions than for the TC1 configuration can be done.
Figure 6: Temperature evolution for the TC2 configuration (TSI: inside surface temperature, TSO: outside surface temperature).
Test TC3: Transient Conduction – Sinusoidal driving temperature

The external surface is exposed to an environment at a 24h periodic sinusoid temperature while the interior zone stays at 20 °C. Results presented in Figure 8 clearly show that both programs give correct temperature evolutions. The greatest errors are found to the 0.25 m concrete wall for BES-2 simulation. Figure 9 presents the effect of decreasing the time step value on the response. Once again, the finite-volume method needs smaller time step than the function transfer method to achieve a good convergence.
Figure 8: Temperature evolution for the TC3 configuration. (TSI: inside surface temperature, TEXT: external temperature).

Figure 9: Temperature evolution for the TC3 configuration (concrete wall) (TSI: inside surface temperature, TS: time step).

MOISTURE TRANSFER RESULTS

Note that even if the vapor concentration is used in the calculation of the analytical solutions, the results are presented here in terms of relative humidity whose typical values are well-known.

Test MT1: Moisture Transfer – Step Response

Figure 10 presents the evolution of the relative humidity in the wall when a sudden increase of relative humidity occurs in the zone. Analytical results are presented
from the wall surface \((x = 0)\) to the effective penetration depth of the BES-1 moisture model \((d_{EMPD} = 0.0066 \text{ m in the present case})\).

The relative humidity at the surface sharply increases and reaches 75 % after 12 hours whereas that at \(x = 0.0066\) equals only 55 % after 12 hours. It should be noted that for the present case, the thickness of the wall (15 cm) can be considered as infinite for this 12 hour period (it takes 300 hours to observe an increase of 1 % at \(x = L\)).

At first sight, BES-1 seems to greatly underestimate the sharp increase of the relative humidity at the surface. It should be pointed out that the result presented for BES-1 is not the relative humidity at the surface but the relative humidity in the surface buffer so its value has to be close to the mean value of the analytical results from \(x = 0\) to \(x = 0.0066\) (+ sign in Figure 10). As shown by the Figure, this value is not exactly the mean value over the surface buffer layer because the deep buffer layer acts as an additional perturbation regarding the surface buffer layer relative humidity.

On the other hand, BES-2 predicts well the evolution with a 1 mm finite-volume length and a 1 min time step value. We performed additional simulations to test the BES-1 accuracy in the first hours after the perturbation. The increase tends to be underestimated at the beginning with the coarsest grids. Increasing the nodes number improve the prediction until 1500 nodes (= 0.1 mm finite-volume length). Refining the grid over 1500 nodes gives negligible improvement for the response.

![Figure 10: Response to a sudden increase of the zone air relative humidity (left: all results, right: BES-2 results for different nodes numbers).](image)

**Test MT2: Moisture Transfer - Periodic Indoor Moisture Perturbation**

Because the purpose of this example is to calculate the relative humidity of the zone air, we performed a preliminary test with impermeable wall internal surfaces to
verify that both programs correctly calculate the vapor mass conservation. Figure 11 presents the evolution of the zone air relative humidity when there is no moisture transfer to the walls. BES-1 predicts well the evolution of the zone air relative humidity with a 15 min time step, using a smaller time step of 3 min leads to the analytical evolution (not shown here). BES-2 perfectly matches the analytical solution.

![Relative Humidity Evolution](image)

Figure 11: Zone air relative humidity evolution without wall buffer storage effect.

Figures 12 and 13 present the relative humidity evolution (within the zone and at the surface) taking the wall buffer storage into account for the three first days and when the periodic state is achieved. The periodic state is reached when the relative difference between two consecutive days relative humidity is lower than 0.01 % for each hour of the 24 hour period.

The left graph of Figure 12 clearly shows that BES-1 does not predict very well the relative humidity variation of the zone air whereas BES-2 gives accurate results. The reason lies in the fact that the moisture transfer in BES-1 is function of the vapor concentration gradient between the air zone and the surface buffer layer and not between the air zone and the wall surface.

At initial time, the vapor concentration in the wall is at its lowest value. As the periodic solicitation goes, the wall vapor concentration will increase until the periodic state is reached (right graph). The real increase given by both the analytical solution and BES-2 is stronger than that obtained with BES-1 as previously observed in Figure 11. As a result the zone air vapor concentration for BES-2 will increase faster than the BES-1 one.

When the periodic state is reached (Figure 13), the zone air vapor concentrations
predicted by both methods are very close (left graph) and close to the analytical solution. It should be noticed that the analytical solution involves some numerical approximations (calculation of the erfc function, finite summations instead of integrals) that can explain some of the differences observed with BES-2.

![Figure 12](image1.png)

Figure 12: Relative humidity evolution with wall buffer storage effect for the three first days (left: zone air, right: at the surface).

![Figure 13](image2.png)

Figure 13: Relative humidity evolution with wall buffer storage effect when the periodic state solution is reached (left: zone air, right: at the surface).

**CONCLUSION**

By simulating cases involving heat or moisture transfer through walls with BES-1 and BES-2, the present study aims to put into relief the advantages and limitations of both methods. The authors are aware that building energy simulation softwares are
different one from each other as they are implemented with different algorithms and convergence schemes, nevertheless general conclusions can be drawn:

- Concerning heat transfer alone, a time step of 15 min for both methods has to be employed to obtain accurate results. Smaller time step values for BES-1 do not always lead to better response.
- Concerning moisture transfer alone, the time step has to be reduced at least to a value of 1 min for BES-2.
- For the tested cases, finite-volume space grid dimension of 1 mm for heat transfer and moisture transfer is needed to fit analytical solutions.
- Considering the previous points, BES-1 is 3 times faster than BES-2 for heat transfer and 80 times faster for moisture transfer. Note that these values have been obtained for the presented simple cases, the simulation time difference between the two methods will increase with the number of walls.

To sum up, both methods give accurate results with reasonable computational time for heat transfer. Predictions with a time step of 1 hour are particularly good in the presence of a thermal sinusoidal solicitation which is usual in real buildings. For moisture transfer, as long as the periodic state remains, i.e. when only the thin external surface layer of the wall is affected by the moisture variation of the surrounding air, BES-1 gives accurate response. On the opposite, when sudden changes occur, BES-1 tends to underestimate the variation whereas BES-2 correctly predicts the moisture evolution in both conditions.

It should be noticed that, in the present study, all material properties have been set constant and that heat and moisture calculations have been decoupled. Moisture dependent thermal conductivity, moisture diffusivity, vapor phase transport coefficient and moisture capacity would modify both heat and moisture transfer. Because these variable properties can not be implemented in a straightforward way into response-factor based methods, finite-volume based methods have to be developed to understand better the coupled heat and moisture transfer that occur within real buildings’ envelope.

ACKNOWLEDGEMENTS

The authors would like to acknowledge J.-P. Deblois, undergraduate student at Ecole Technologique Supérieure at Montreal (Canada), for his helpful participation and dedication regarding the heat transfer test cases during its four-month period at PUC-PR. The authors thank CNPq – Conselho Nacional de Desenvolvimento Científico e Tecnológico – of the Secretary for Science and Technology of Brazil.
REFERENCES


APPENDIX A
Analytical Solution for Step Changes in External Air Temperature with Adiabatic Internal Surface.

The analytical solution is based on the one-dimensional unsteady heat conduction equation:

$$\frac{\partial T_w(x,t)}{\partial t} = a \frac{\partial^2 T_w(x,t)}{\partial x^2}.$$  

The boundary conditions are:

- $T_w(x,0) = T_{out}(0) = T_0$,
- $T_{out}(t > 0) = T_1$,
- $\frac{\partial T_w(x = 0, t > 0)}{\partial x} = 0$, and
- $h(T_w(x = L, t) - T_{out}) = -\lambda \frac{\partial T_w(x = L, t)}{\partial x}$.

The solution is given by Incropera and DeWitt (1990):

$$T_w(x,t) = T_1 + (T_0 - T_1) \sum_{n=1}^{\infty} a_n e^{-\alpha^2 Fo} \cos\left(\frac{\alpha x}{L}\right).$$

where $\alpha \tan \alpha = Bi$, $Bi = \frac{h_{in}L}{\lambda}$ and $Fo = \frac{at}{L^2}$.

APPENDIX B
Analytical Solution for Step Changes in External Air Temperature when the Inside Air Temperature is held Constant.

The analytical solution is based on the one-dimensional unsteady heat conduction equation:

$$\frac{\partial T_w(x,t)}{\partial t} = a \frac{\partial^2 T_w(x,t)}{\partial x^2}.$$  

The boundary conditions are:

- $T_w(x,0) = T_{in}(0) = T_{out}(0) = T_0$,
- $T_{in}(t > 0) = T_0$,
- $T_{out}(t > 0) = T_1$,
- $h_{in}(T_w(x = 0, t) - T_{in}) = -\lambda \frac{\partial T_w(x = 0, t)}{\partial x}$, and
- $h_{out}(T_{out} - T_w(x = L, t)) = -\lambda \frac{\partial T_w(x = L, t)}{\partial x}$.

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The solution is given by Özisik (1980):

$$\theta(X,Fo) = Bi_{out} \sum_{n=1}^{\infty} \left[ \alpha_n \cos(\alpha_n X) + Bi_{out} \sin(\alpha_n X) \right] \frac{1-e^{-\alpha_n^2 t}}{\alpha_n N(\alpha_n)}.$$

where \(\theta(X,t) = \frac{T_w(x,t) - T_0}{T_1 - T_0}\), \(\tan \alpha_n = \frac{\alpha_n (Bi_{in} + Bi_{out})}{\alpha_n^2 - Bi_{in} Bi_{out}}\),

$$2N(\alpha_n) = \left[ \alpha_n^2 + Bi_{out}^2 \right] \times \left[ 1 + \frac{Bi_{in}}{\alpha_n^2 + Bi_{in}^2} \right] + Bi_{out},$$

$$Bi_{in} = \frac{h_{in} L}{\lambda}, \ Bi_{out} = \frac{h_{out} L}{\lambda}, \ Fo = \frac{at}{L^2} \ \text{and} \ X = \frac{x}{L}.$$

**APPENDIX C**

**Analytical Solution for Sinusoidal Changes in External Air Temperature when the Inside Air Temperature is held Constant.**

The analytical solution is based on the one-dimensional unsteady heat conduction equation:

$$\frac{\partial T_w(x,t)}{\partial t} = a \frac{\partial^2 T_w(x,t)}{\partial x^2}.$$

The boundary conditions are:

- \(T_w(x,0) = T_0\),
- \(T_{in}(t > 0) = T_0\),
- \(T_{out}(t > 0) = T_0 + T_{amp} \sin \left( \frac{2\pi t}{\tau} \right)\),
- \(h_{in}(T_w(x = 0, t) - T_{in}) = -\lambda \frac{\partial T_w(x = 0, t)}{\partial x}\), and
- \(h_{out}(T_{out} - T_w(x = L, t)) = -\lambda \frac{\partial T_w(x = L, t)}{\partial x}\).

The solution is given by Spitler et al. (2001):

$$q_{is}(t) = \frac{f}{R} \times T_{amp} \times \sin \left( \frac{2\pi (t - \phi)}{\tau} \right), \ \text{and}$$

$$T_{is}(t) = T_0 + \frac{q_{is}(t)}{h_{in}}.$$
where f = \left| \frac{R}{m} \right|, \phi = \frac{12}{\pi} \arctan \left( \frac{\text{Im}(R/m)}{\text{Re}(R/m)} \right), \quad m = \frac{\cosh(p)}{h_{\text{out}}} + \frac{\sinh(p)}{k \cdot p} L, \quad p = \left( \frac{\pi L^2}{\alpha \tau} \right)^{0.5} \times (1 + i), \quad \text{and} \quad R = \frac{1}{h_{\text{in}}} + \frac{1}{h_{\text{out}}} + \frac{L}{k}.

**APPENDIX D**

**Semi-Analytical Solution for Room with Buffering Effect**

The analytical solution is based on the one-dimensional unsteady moisture diffusion equation:

$$\frac{\partial c_w(x,t)}{\partial t} = a_c \frac{\partial^2 c_w(x,t)}{\partial x^2}.$$  

The wall boundary conditions are:

- $c_w(x,0) = c_{\text{in}}(0) = c_{\text{out}}(0) = c_0$,
- $c_{\text{out}}(t > 0) = c_0$,
- $\frac{\partial c_w(x = L, t > 0)}{\partial x} = 0$, and
- $\beta_c(c_w(x = 0, t) - c_{\text{in}}) = -\delta_c \frac{\partial c_w(x = 0, t)}{\partial x}$.

where $a_c = \frac{\delta_c c_{\text{sat}}}{\zeta}$, $\beta_c = \beta RT$ and $\delta_c = \delta_p RT$.

The balance equation for the room air is:

$$V \frac{\partial c_{\text{in}}(x,t)}{\partial t} = Q_v (c_{\text{out}} - c_{\text{in}}(x,t)) - A \beta_c (c_{\text{in}}(x,t) - c_w(x,t)) + G.$$  

Considering a constant external water vapor concentration and that the moisture capacity of the room air is negligible, a semi-analytical solution based on superposition of solutions at each time step and Laplace transforms is given by Bednar and Hagentoft (2005):

$$c_{\text{in}}(x,t) = c_{\text{out}} + \sum_{n=1}^{\infty} c_n(t).$$

where $c_n(t) = \frac{G_{0,n}}{Q_v} u(t - t_{0,n}) \left( 1 - \left( 1 - \frac{d_{\text{s}}}{d_2} \right) e^{-\frac{a_{c,n}}{d_2}(t-t_{0,n})} \right) \text{erfc} \left( \frac{a_{c,n}}{\sqrt{d_2^2 (t-t_{0,n})}} \right),$
\[ d_2 = d_s + \frac{A\delta_c}{Q_v}, \quad d_s = \frac{\delta_c}{\beta_c}, \quad \text{and} \]

\[ G_{0,n} \text{ and } t_{0,n} \text{ defined by:} \]

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>( n=2m )</th>
<th>( n=2m+1 )</th>
</tr>
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<tbody>
<tr>
<td>( t_{0,n} )</td>
<td>9</td>
<td>17</td>
<td>9+24m</td>
<td>17+24m</td>
</tr>
<tr>
<td>( G_{0,n} )</td>
<td>+ G_0</td>
<td>- G_0</td>
<td>+ G_0</td>
<td>- G_0</td>
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# NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>thermal diffusivity</td>
<td>( m^2.s^{-1} )</td>
</tr>
<tr>
<td>A</td>
<td>wall surface area</td>
<td>( m^2 )</td>
</tr>
<tr>
<td>( a_c )</td>
<td>moisture diffusivity for vapor concentration</td>
<td>( m^2.s^{-1} )</td>
</tr>
<tr>
<td>Bi</td>
<td>Biot number</td>
<td>-</td>
</tr>
<tr>
<td>c</td>
<td>water vapor concentration</td>
<td>( kg.m^{-3} )</td>
</tr>
<tr>
<td>( c_m )</td>
<td>specific heat</td>
<td>( J.kg^{-1}.K^{-1} )</td>
</tr>
<tr>
<td>( D_{0v} )</td>
<td>vapor phase transport coefficient for moisture volumetric content</td>
<td>( m^2.s^{-1} )</td>
</tr>
<tr>
<td>( d_{EMPD} )</td>
<td>effective moisture penetration depth</td>
<td>( m )</td>
</tr>
<tr>
<td>f</td>
<td>decrement factor</td>
<td>-</td>
</tr>
<tr>
<td>( f(\phi, \omega) )</td>
<td>conversion factor relative humidity to humidity ratio</td>
<td>-</td>
</tr>
<tr>
<td>Fo</td>
<td>Fourier number</td>
<td>-</td>
</tr>
<tr>
<td>h</td>
<td>convective heat transfer coefficient</td>
<td>( W.m^{-2}.K^{-1} )</td>
</tr>
<tr>
<td>L</td>
<td>wall layer thickness</td>
<td>( m )</td>
</tr>
<tr>
<td>( L_{vap} )</td>
<td>heat of vaporization</td>
<td>( J.kg^{-1} )</td>
</tr>
<tr>
<td>M</td>
<td>mass of material</td>
<td>( kg )</td>
</tr>
<tr>
<td>q</td>
<td>heat flux</td>
<td>( W.m^{-2} )</td>
</tr>
<tr>
<td>( Q_v )</td>
<td>volumetric airflow rate</td>
<td>( m^3.s^{-1} )</td>
</tr>
<tr>
<td>R</td>
<td>gas constant of water vapor</td>
<td>( J.kg^{-1}.K^{-1} )</td>
</tr>
<tr>
<td>( R )</td>
<td>overall thermal resistance of the wall</td>
<td>( m^2.K/W )</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
<td>( s )</td>
</tr>
<tr>
<td>T</td>
<td>absolute temperature</td>
<td>( K )</td>
</tr>
<tr>
<td>u</td>
<td>step function</td>
<td>-</td>
</tr>
<tr>
<td>V</td>
<td>volume of the room air</td>
<td>( m^3 )</td>
</tr>
<tr>
<td>x</td>
<td>distance in the wall layer</td>
<td>( m )</td>
</tr>
<tr>
<td>X</td>
<td>dimensionless distance in the wall layer</td>
<td>-</td>
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Greek symbols

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<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>( \alpha )</td>
<td>root of the transcendental equation</td>
<td>rad</td>
</tr>
<tr>
<td>( \beta )</td>
<td>surface transfer coefficient for humidity ratio</td>
<td>( kg.s^{-1} )</td>
</tr>
<tr>
<td>( \beta_c )</td>
<td>surface transfer coefficient for vapor concentration</td>
<td>( m.s^{-1} )</td>
</tr>
<tr>
<td>( \delta_c )</td>
<td>moisture permeability for vapor concentration</td>
<td>( m^2.s^{-1} )</td>
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<tr>
<td>( \delta_p )</td>
<td>moisture permeability for partial vapor pressure</td>
<td>( kg.m^{-1}.s^{-1}.Pa^{-1} )</td>
</tr>
<tr>
<td>( \phi )</td>
<td>time lag</td>
<td>( h )</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>thermal conductivity of the dry material</td>
<td>( W.m^{-1}.K^{-1} )</td>
</tr>
<tr>
<td>( \theta )</td>
<td>moisture volumetric content</td>
<td>( m^3.m^{-3} )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density of the dry material</td>
<td>( kg.m^{-3} )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>time period</td>
<td>( s )</td>
</tr>
<tr>
<td>( \omega )</td>
<td>humidity ratio</td>
<td>( kg.kg^{-1} )</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>moisture capacity of the material</td>
<td>( kg.m^{-3} )</td>
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Subscripts

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<th>Description</th>
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<tbody>
<tr>
<td>0</td>
<td>initial (at t=0)</td>
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<tr>
<td>amp</td>
<td>amplitude</td>
</tr>
<tr>
<td>in</td>
<td>indoor</td>
</tr>
<tr>
<td>is</td>
<td>internal surface</td>
</tr>
<tr>
<td>out</td>
<td>outdoor</td>
</tr>
<tr>
<td>sat</td>
<td>saturation</td>
</tr>
<tr>
<td>w</td>
<td>wall</td>
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